

Isospin Decomposition of the Photoproduced $\Sigma\pi$ System Near the $\Lambda(1405)$

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Abstract

Recent experimental results for the reaction $\gamma + p \rightarrow K^+ + \Sigma + \pi$ from CLAS at Jefferson Lab are discussed. It was found that the mass distributions or “line shapes” of the three charge combinations $\Sigma^+\pi^-$, $\Sigma^0\pi^0$ and $\Sigma^-\pi^+$ differ significantly. Our results show that the $\Lambda(1405)$, as the $I = 0$ constituent of the reaction, must be accompanied by an $I > 0$ component. We discuss phenomenological fits to the data to test the possible forms and magnitudes of these amplitudes. A two-amplitude $I = 0$ fit of Breit-Wigner form to the $\Sigma^0\pi^0$ channel alone works quite well. The addition of a single $I = 1$ amplitude works fairly well to model all the line shapes simultaneously.

Keywords: hyperon, photoproduction, line shape, $\Lambda(1405)$

1. Introduction

The CLAS Collaboration at Jefferson Lab has measured [1], as part of a large program of photoproduction reactions in the GeV energy region, the channel $\gamma + p \rightarrow K^+ + \Sigma + \pi$. The $\Sigma\pi$ invariant masses range was from very close to threshold at 1328 MeV to well beyond 1520 MeV where the $\Lambda(1520)$ is prominently produced. In between these limits, the $\Lambda(1405)$ is produced, as well as the $\Sigma(1385)$ and perhaps other possible states. The CLAS experiment is the first to simultaneously map out this mass range in all three charge combinations $\Sigma^+\pi^-$, $\Sigma^0\pi^0$ and $\Sigma^-\pi^+$. This is interesting because comparison can reveal (1) whether there is substantial contribution of $I > 0$ amplitudes to the reaction mechanism even after the $\Sigma(1385)$ is removed, and (2) how strong the channel coupling is between the $\Sigma\pi$ final state and the competing $N\bar{K}$ final state. Thus, while there have been hints of line shape distortions of the $\Lambda(1405)$ in earlier experiments [2–8] the CLAS experiment offers the best look so far at this phenomenon.

Satisfactory theoretical understanding of the $\Lambda(1405)$ continues to be a challenge. For a recent review, see Ref [9]. For example, within the spectrum of P-wave baryons within the relativistic quark model it is a poor quantitative fit [10, 11]. In models of so-called chiral unitary state generation among octet baryons and pseudo-scalar mesons, this state plays an important

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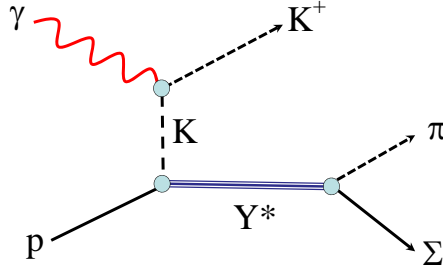


Figure 1: (Color online) Creation of the three-body $K^+ \Sigma \pi$ final state via an intermediate hyperon in the reaction $\gamma + p \rightarrow K^+ + \Sigma + \pi$. In this particular example, a t -channel exchange enables an off-shell kaon to create a $\Lambda(1405)$ that is sub-threshold for on-shell $N\bar{K}$ reactions.

role [12, 13]. It arises through channel-coupling of the $N\bar{K}$ system, with threshold at 1432 MeV, and the $\Sigma\pi$ system into which the $\Lambda(1405)$ must decay. In the flavor symmetric SU(3) limit, chiral unitary models show [14] that two new octets and a singlet of $J^P = \frac{1}{2}^-$ states are formed which, when SU(3) is broken, leads to the presence of two $I = 0$ poles in the hyperon sector that reside near the mass of the $\Lambda(1405)$. Meanwhile, the $I = 1$ poles vanish or are very weak. Thus, it is of interest to see how well the “two-pole” picture of the $\Lambda(1405)$ is supported by the data. One prediction is that the line shape of the $\Lambda(1405)$ depends on the reaction channel through which it is excited, since a given reaction can couple with differing strengths to the two predicted poles. In the present case we look using real photoproduction off a proton target. This is illustrated in Fig. 1 which shows how an off-shell kaon exchange can create the $\Lambda(1405)$ that is sub-threshold for a free kaon plus a nucleon. A calculation of this channel was reported in Ref. [15] and compared to our results in Ref. [1].

The CLAS measurements are described in detail in Ref. [1]. Briefly, tagged, real, unpolarized photons [16] with a highest energy of 3.8 GeV impinged upon a 40 cm liquid hydrogen target. About 20 billion events with at least two charged tracks in the large-acceptance toroidal spectrometer [17] were recorded, and in the analysis about 1.41×10^6 events with $K^+\Sigma\pi$ were isolated. Charged kaons, protons and pions were identified by time of flight over a four to five meter flight path.

The Σ^+ was reconstructed via both of its decay modes, a proton with an undetected π^0 , or a π^+ with an undetected neutron. The Σ^- was reconstructed via its decay to π^- and an undetected neutron. Regions of kinematic overlap of the two charged Σ ’s were cut away. The $\Sigma^0\pi^0$ final state was reconstructed through its decay via $\Lambda \rightarrow \pi^- p$ with an undetected π^0 and photon. The undetected particles were identified via missing mass, and a 1C kinematic fit was employed to optimize resolution. The exception was the $\Sigma^0\pi^0$ case where two neutral particles were missing, so the selection was made using a fixed range of missing masses.

The two identified backgrounds removed were from $K^+\Sigma^0(1385)$ production and from $K^*\Sigma^0$ production. The first of these was measured directly in its dominant decay mode to $\Lambda\pi^0$ in P-wave. Using the 11.7% branching fraction to $\Sigma^\pm\pi^\mp$ we could accurately subtract this contribution; this was generally a small fraction of the total $\Sigma\pi$ data. Coherent interference with S-wave decays was in principle canceled due to the full angular coverage of the $\Sigma\pi$ final state in the Y^* rest frame. $K^*\Sigma$ production resulted in a broad background underneath the whole of the $\Sigma\pi$ spectrum, with varying kinematic overlap in different bins of W . Studies showed that making increasingly drastic rejection cuts of events in the wide K^* region made no change in the line shapes of the $\Sigma\pi$ results. We were unable to find any hint of coherent interference due to this background.

Both backgrounds were modeled in a Monte Carlo simulation and subtracted away. Details of the analysis procedures, including systematic studies, can be found in Refs. [1] and [18].

The analysis used 100 MeV wide bins in γp center of mass energy W to present the results, and it will be seen that the line shapes differ as W increases.

In this article we present two fits to the $\Sigma\pi$ line shape data that are different from the “best” overall fit described in Ref. [1]. In particular, we first show the fit obtained when allowing two $I = 0$ Breit-Wigner (BW) amplitudes to fit only the $\Sigma^0\pi^0$ data. This final state cannot be reached via an $I = 1$ decay by reason of the isospin-addition coefficients, and we neglect $I = 2$ processes. Thus, this fit can serve to test the two-pole picture of the $\Lambda(1405)$. Second, we present a fit that adds a single $I = 1$ BW amplitude to the previous two $I = 0$ amplitudes and use this to fit all three $\Sigma\pi$ states at once. This serves to characterize the isovector piece of the interaction needed to represent the experimental data. This amplitude combination leads to a result that is statistically less good than the one described in Ref. [1] which used one $I = 0$ amplitude and two $I = 1$ amplitudes. On the other hand, it is closer to some theoretical biases about the likely two-pole structure of the $\Lambda(1405)$. Section 2 outlines the fit procedure, and Section 3 discusses the results.

2. Model for Mass Distributions

As described in Ref. [1], we posit that the $\Sigma\pi$ part of the three-body K^+, Σ, π final states arises via an interaction schematically labeled $\hat{T}^{(I)}$, where I denotes the isospin of the $\Sigma\pi$ system. Let the matrix element squared be denoted by

$$|t_I|^2 \equiv |\langle I, I_3 = 0 | \hat{T}^{(I)} | \gamma p \rangle|^2. \quad (1)$$

The matrix element expressions that give the three measured final state charge combinations are then given by the coherent sum of the appropriately Clebsch-Gordon-weighted t_I elements as

$$|T_{\pi^-\Sigma^+}|^2 = \frac{1}{3}|t_0|^2 + \frac{1}{2}|t_1|^2 - \frac{2}{\sqrt{6}}|t_0||t_1|\cos\phi_{01}, \quad (2)$$

$$|T_{\pi^0\Sigma^0}|^2 = \frac{1}{3}|t_0|^2, \quad (3)$$

$$|T_{\pi^+\Sigma^-}|^2 = \frac{1}{3}|t_0|^2 + \frac{1}{2}|t_1|^2 + \frac{2}{\sqrt{6}}|t_0||t_1|\cos\phi_{01}. \quad (4)$$

At a fixed γp center of mass energy W the matrix elements are parametrized as relativistic Breit-Wigner functions $B_I(m)$, where m is the $\Sigma\pi$ mass, with an overall strength $C(W)$ and a strong production phase $\Delta\phi_I$ as

$$t_I(m) = C_I(W)e^{i\Delta\phi_I}B_I(m). \quad (5)$$

The Breit-Wigner amplitude form is

$$B_I(m) = \sqrt{\frac{2}{\pi}} \left[\frac{\sqrt{mm_R}\Gamma_I^0 \left(\frac{q}{q_R}\right)^{2L}}{m_R^2 - m^2 - im_R\Gamma_{\text{tot}}(q)} \right] \quad (6)$$

where m_R is the centroid of the resonance distribution, Γ_I^0 is the fixed decay width to a given final state, and $\Gamma_{\text{tot}}(q)$ is the total width to all final states. The available momentum in the decaying

hyperon center of mass system is q , and in this frame q_R is the available decay momentum at $m = m_R$. Angular momentum L is zero in the present case of an S-wave decay of the odd-parity $\Lambda(1405)$ decaying to a pseudo-scalar meson and an octet baryon.

The mass-dependent width in the denominator was treated in a way that accounts for channel coupling using the Flatté prescription [19]. That is, if decay mode “1” is the $\Sigma\pi$ final state, and decay mode “2” is the $N\bar{K}$ final state, then part of the mass range is below threshold for mode 2. Nevertheless, mode 2 has influence on mode 1 both below and above threshold. To preserve unitarity and the analytic form of the amplitude, we analytically continue the center of mass momentum q of decay mode 2 to below its threshold. We write

$$\Gamma_{\text{tot}}(m) = \Gamma_{I,1}(q_1(m)) + \Gamma_{I,2}(q_2(m)), \quad (7)$$

where the decay channels are described by width

$$\Gamma_{I,j}(q) = \Gamma_{I,j}^0 \frac{m_R}{m} \left(\frac{q_j(m)}{q_R} \right)^{2L+1}. \quad (8)$$

Below mode 2 threshold m_{thresh} , the momentum $q_2(m)$ is nominally zero. However, we continue the momentum to imaginary values for $m < m_{\text{thresh}}$. Furthermore, we introduce a Flatté factor for the branching fraction of the decay modes as

$$\gamma = \Gamma_{I,2}^0 / \Gamma_{I,1}^0. \quad (9)$$

This factor is determined by the fits, and indicates the ratio of the channel coupling.

Finally, the mass distribution is computed as a cross section differential in $\Sigma\pi$ mass m by including all the flux and phase space factors for the initial and final states as

$$\frac{d\sigma_{ab}}{dm} = \frac{(\hbar c)^2 \alpha}{64\pi^3} \frac{p_{K^+} q}{p_{\gamma p} W^2} |T_{\pi^a \Sigma^b}|^2. \quad (10)$$

Here p_{K^+} is the c.m. kaon momentum and $p_{\gamma p}$ is the initial state c.m. momentum. We have factored out the electromagnetic coupling strength α so that $C(W)$ becomes an effective strong coupling in units of $\sqrt{\text{GeV}}$. This expression is integrated over all kaon production angles, a step that was necessary to have adequate statistics. Similarly, it is integrated over all Σ decay angles in the hyperon rest frame, as mentioned, to ensure cancellation of any interference with P-wave decays from the $\Sigma(1385)$ (which is anyway known to be small and was incoherently subtracted).

3. Results

First we consider the fit obtained when using two $I = 0$ amplitudes fitted to the data for $\Sigma^0\pi^0$ only. This decay mode should be dominated by the $I = 0$ amplitude, assuming $I = 2$ can be neglected. Figure 2 shows a sample result in one 100 MeV wide bin of W at 2.3 GeV. The solid blue curve shows the total fit, and the solid and dashed black curves show the cross section that would result from each of the two amplitudes separately. Also shown is a linear background (not part of the fit) from threshold to 1.6 GeV that was included to account for the imperfect modeling of background in the experimental analysis. Data points near the location of the $\Lambda(1520)$ position have also been suppressed. The fit was done not just to the W bin shown, but to all nine bins at once. The full data set is shown in Fig. 3. In each panel the black curves have the same shape, differing only in overall weight. The weights, the $C_I(W)$, are shown in Fig. 4.

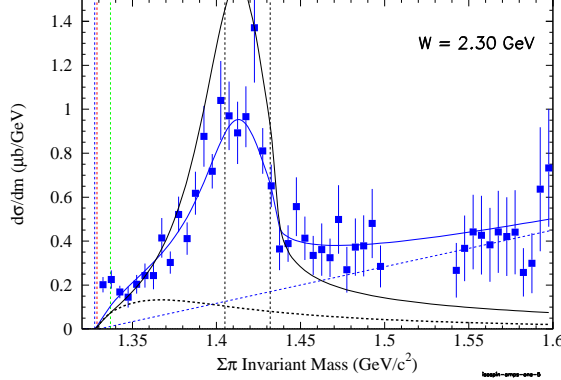


Figure 2: (Color online) Two $I = 0$ amplitudes fitted to $\Sigma^0\pi^0$ only (solid and dashed black curves). The error bars are combined statistical and systematic point-to-point uncertainties. The solid blue curve shows the fit to the $\Sigma^0\pi^0$ data (blue points). The three vertical dotted lines at the left are thresholds for the three decay modes, and vertical black dotted lines mark the nominal 1.405 GeV mass and the location of the $N\bar{K}$ threshold m_{thresh} . The incoherent background is shown as a thin dashed line (blue).

It is evident visually that the fit is good; the overall reduced χ^2 of the fit was 0.89. The results of the fit are given in Table 1. The two $I = 0$ amplitudes have centroids at 1329 and 1390 MeV, respectively, with respective widths that both differ when compared to the 50 MeV RPP value for the $\Lambda(1405)$ width. This is in part explained by the action of the channel coupling. It is seen in the table that the Flatté factor for the narrower amplitude (the only one permitted in the fit) is large, at 1.5. This has the effect of distorting the line shape around the $N\bar{K}$ threshold, as seen in the figures, but it also has the effect of making the most probable mass somewhat above 1405 MeV, even though the nominal centroid is at 1329. The goodness of this fit may be taken as positive evidence for the two-pole solution for the $I = 0$ state(s). Figure 4 shows how the two components of the fit vary in strength as W increases: evidently the broader one fades out quickly while the dominant narrower one remains strong. We have no interpretation to offer for this behavior, nor for the relative production phase between the two amplitudes given in the table.

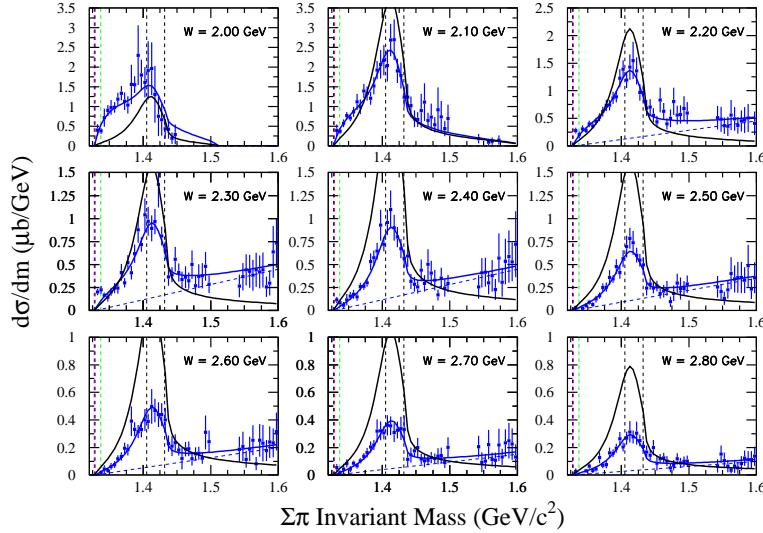


Figure 3: (Color online) Two $I = 0$ amplitudes fitted to $\Sigma^0\pi^0$ only. Data and fits for $\Sigma^0\pi^0$, with each panel showing the indicated value of W . Data and curve shapes and colors are as in Fig. 2. The curves in all panels use the same amplitude parameters from Table 1.

Table 1: Results of the fit using two $I = 0$ Breit-Wigner line shapes fitted to only the $\Sigma^0\pi^0$ channel. The uncertainties reflect the stability of repeated fits with varying initial values. “N/A” means no free parameter was allowed.

Amplitude	Centroid m_R (MeV)	Width Γ_0 (MeV)	Phase $\Delta\Phi_I$ (radians)	Flatté γ Factor
$I = 0$	1329 ± 10	20 ± 10	N/A	1.5 ± 0.3
$I = 0$	1390 ± 10	174 ± 20	-0.2 ± 0.3	N/A

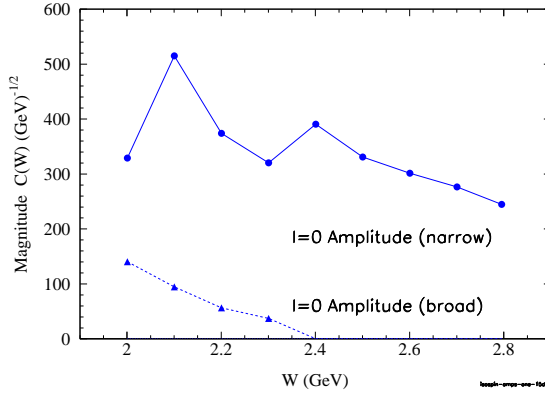


Figure 4: (Color online) For two $I = 0$ amplitudes fitted to $\Sigma^0\pi^0$ only, the figure shows the strength of each of the isospin amplitudes, $C_0(W)$, as a function of W , per Eq. 5.

Next we consider the best fit obtained when using two $I = 0$ amplitudes and a single $I = 1$ amplitude fitted to the data for all three charge final states: $\Sigma^+\pi^-$, $\Sigma^0\pi^0$ and $\Sigma^-\pi^+$. The fit parameters are given in Table 2; the reduced χ^2 for this fit was 2.60. Figure 5 shows a sample result in one 100 MeV wide bin of W at 2.3 GeV. The colored curves show the total fits to each charge state, and the solid, dashed, and dotted black curves show the cross sections that would arise from each of the three amplitudes separately. The dotted black curve shows the $I = 1$ piece of the reaction. Its width is similar to the others (54 MeV), with a centroid at 1367 MeV, and a Flatté factor that is close to unity, like the one for the $I = 0$ piece. The colored curves show fairly successful modeling of the split between the three charge combinations.

Table 2: Results using two $I = 0$ and one $I = 1$ Breit-Wigner line shapes and fitting to all final charge states simultaneously. The uncertainties reflect the stability of repeated fits with varying initial values. “N/A” means no free parameter was allowed.

Amplitude	Centroid m_R (MeV)	Width Γ_0 (MeV)	Phase $\Delta\Phi_I$ (radians)	Flatté γ Factor
$I = 0$ (low mass)	1338 ± 10	44 ± 10	N/A	0.94 ± 0.20
$I = 0$ (high mass)	1384 ± 10	76 ± 10	1.8 ± 0.4	N/A
$I = 1$	1367 ± 20	54 ± 10	2.2 ± 0.4	1.19 ± 0.20

Again, the fit was done not just to the W bin shown, but to all nine bins at once. The full data

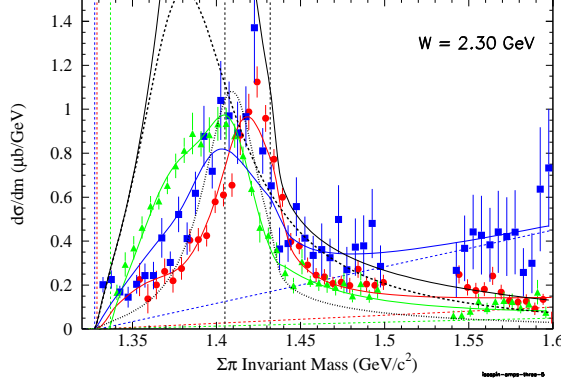


Figure 5: (Color online) Fits of the isospin amplitudes $I = 0$ (solid and dashed black curves) and $I = 1$ (dotted curve) at the given value of W . Data and fitted curves for the three charge states are: red curve and circles $\Sigma^+\pi^-$, blue curve and squares: $\Sigma^0\pi^0$, green curve and triangles: $\Sigma^-\pi^+$. The error bars are combined statistical and systematic point-to-point uncertainties. The three vertical dotted lines at the left are thresholds for the respective decay modes, and vertical dotted lines mark the nominal 1.405 GeV mass and the location of the $N\bar{K}$ threshold m_{thresh} . The incoherent background is shown as the thin dashed lines.

set is shown in Fig. 6. In each panel the black curves have the same shapes, differing only in overall weight. The way in which the normalizations $C_I(W)$ of the three amplitudes vary with W is shown in Fig. 7. Note that it is the square of these coefficients that fixes the relative strength of the contributions. The dominant contribution comes from the $I = 0$ amplitude, as expected. The $I = 1$ amplitude is weaker in magnitude by a factor of about three, and is about equal in size to the smaller of the two $I = 0$ amplitudes.

It is of interest to compare Table 2 with the previous simpler case shown in Table 1. Upon adding the interference-causing $I = 1$ amplitude, the position and width of the dominant $I = 0$ amplitude rises from 1327 MeV to 1338 MeV; both are close to $\Sigma\pi$ threshold, and agree within the uncertainties. The Flatté factor remains in the vicinity of 1.0. The weaker $I = 0$ amplitude stays at the same mass, within uncertainties, and gets narrower. Finally, one sees that the $I = 1$ amplitude has a centroid at 1367 MeV with a width of 54 MeV. It had a fitted Flatté factor as well, and the fit placed it also close to unity.

Choosing the $I = 1$ amplitude to have a resonant Breit-Wigner form was, of course, an assumption made to create a model. Alternatively one might construct a non-resonant scattering amplitude with isovector character. But if this analysis is accurate, and if it can be confirmed that an $I = 1$ component in the reaction is reasonably fit with a Breit-Wigner resonance-like structure, it raises the question of how to interpret the result. It may point, for example, to the existence of a low mass $J^P = \frac{1}{2}^- \Sigma^*$ state as discussed in Ref. [20]. A full theoretical treatment is now needed.

4. Conclusions

The model of the mass distribution for the $\Sigma\pi$ line shape introduced in Ref. [1] has been described briefly. It has been applied to recent photoproduction data from CLAS in two alternative combinations in order to further explore possible interpretation of the experimental results.

It has been shown that the $I = 0$ contribution was qualitatively consistent with the picture of two poles, modeled as two Breit-Wigner resonances with large channel coupling to the $N\bar{K}$ final state. It has been shown, moreover, that an $I = 1$ component of the reaction mechanism can give a reasonable explanation of the variation among charge states in the $\Sigma\pi$ system seen in photoproduction. This component was also modeled as a Breit-Wigner line shape with Flatté coupling to the $N\bar{K}$ final state. As shown in Table 2, this $I = 1$ strength is centered near 1367

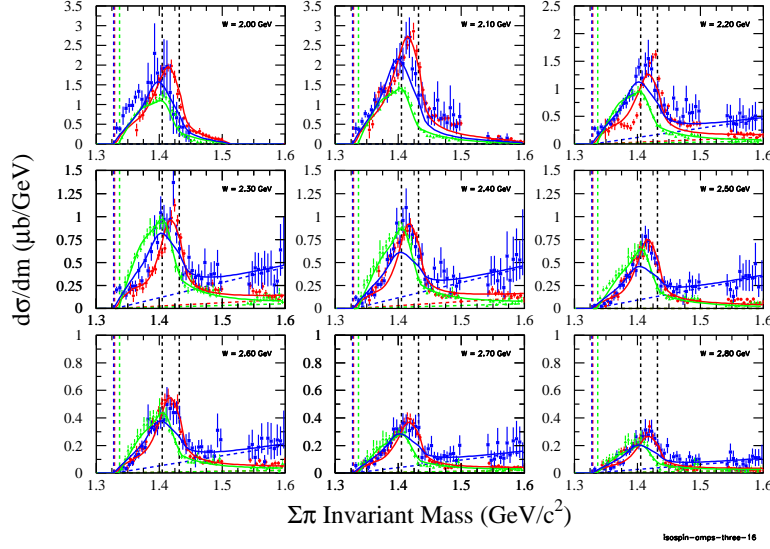


Figure 6: (Color online) Fits to two $I = 0$ amplitudes and one $I = 1$ amplitude, with each panel showing the indicated value of W , without the underlying pure isospin states shown. Data and curve shapes and colors are as in Fig. 5.

MeV. If this is the correct way to model this component of the reaction mechanism, it would lend support to existence of a low mass Σ^* state with $J^P = \frac{1}{2}^-$.

Nevertheless, it must be pointed out that a better fit (in the sense of both χ^2 and the visual appearance of the fit results) was found [1] previously with an alternative choice of one $I = 0$ amplitude and two $I = 1$ amplitudes. The χ^2 in that study was 2.15. Neither is the model unique, nor is the data precise enough to allow only a unique result from the given model. Clearly, more modeling and more theoretical work on the interpretation should follow.

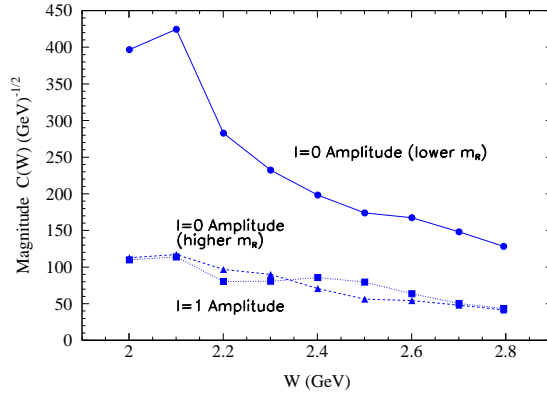


Figure 7: (Color online) For two $I = 0$ amplitudes and one $I = 1$ amplitude, the figure shows the strength of each of the isospin amplitudes, $C_I(W)$ as a function of W , per Eq. 5.

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